Neste artigo, formalizamos o conceito de impunidade num mercado com crimes. Usamos este conceito para precificar crimes que são punidos com multas. Também consideramos um sistema legal como sendo um conjunto de leis estabelecidas no Código Penal determinando as multas no mercado com crimes. Um sistema legal é dito incompleto se o número de leis para cada crime cometido é menor que a incerteza representada pelo número finito de estados da natureza. Assumindo que o sistema legal é incompleto e que não existe impunidade no mercado com crimes, obtemos os seguintes resultados: primeiro, o problema do criminoso sempre tem solução. Segundo, o valor esperado da multa, imposto pelo Judiciário, deve ser o valor futuro do ganhos ilícitos. Terceiro, concluímos que, num contexto de equilíbrio geral, os formuladores de políticas públicas devem fazer um esforço para aumentar o valor esperado das multas, que podem ser atingidas com aplicações rigorosas da lei, aumento do número de policiais nas ruas e redução dos custos judiciais.

**Palavras-chave**: Impunidade; Crime; Incerteza.

**Keywords**: Impunity; Crime; Uncertainty.

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1. Introduction

Impunity seems to be ubiquitous in many countries, particularly in those ones where the corruption is disseminated into power groups. On the one hand, impunity triggers more crimes, which in turn affect the productive sector of the economy by leading to lower economic growth, for an ampler discussion on this theme, see Bardhan (1997). On the other hand, when the crimes are not punished, the corruption increases in the society provoking negative impacts on the economic development, see Mauro (1995).

Our main concern in this paper is to use the economic approach, in particular the choice theory, in order to approach the criminal behaviour. Since the economic approach is without doubt one of the most powerful tools used to analyse and understand the behaviour of individuals in society in particular, those who are engaged in illegal activities, we use it to analyse the offense markets in absence of impunity. This fact was recognised by Becker (1960) in his pioneering work on crime. He was followed by Stigler (1970) and Ehrlich (1973). These three researchers laid the foundation for the economics of crime. Since then, the theoretical and empirical economic literature on crime has grown rapidly; see Merlo (2004) for more literature. For a more recent survey on the economics contributions to the understanding of crime, we refer to Levitt and Miles (2007); and to understand its determinants, see Imrohoroglu, Merlo and and Rupert (2006).

Our model is related to several streams of the literature on crime. The economic models of crime have been categorised either portfolio problems, in which the agent must decide how much wealth to put at risk through involvement in crime, or labor supply problems, in which the agent must choose the amount of time to be allocated to illegal activity, see Heineke (1978). Our paper fits the first category best, where the works of Allingham and Sandmo (1972), Kolm (1973), and Singh (1973), stand out. Examples of the labor supply problem include Becker (1968), Ehrlich (1973), Sjoquist (1973), and Block and Heineke (1975), and Davis (1988). However, the closer paper to our in terms of impunity and not in methodological terms is that of Gordon, Iglesias, Semeshenko and Nadal (2009).

The approach used in this paper is purely financial and is concerned with the decision that criminals take in an uncertain environment. As said before, individuals in our model are involved in illegal activities. The fundamental characteristic of an illegal activity is that its fruits can be savored before the cost of their acquisition must be paid. This characteristic allows criminals to transfer, in an illegal way\(^3\) wealth from the future to the present, enabling them to increase their consumption. Society punishes these activities after they are committed. So the punishment, in terms of fines, will be contingent to the states of nature. Any individual who commits a crime will want to enjoy the returns of the crime without suffering any punishment.

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\(^3\) A legal way of transferring wealth is via selling of assets in financial markets.
We will see that this situation, called impunity, will not be compatible with the agent's utility-maximiser behaviour. That is, under impunity the individual's problem has no solution.

The main objective of this paper is to study the problem of a criminal under the assumption of non-impunity in the markets for offenses. We define impunity as a situation in which the criminal obtains positive returns with certainty in the initial period and with no punishment at the end of the second period. Our analysis is carried out under the assumption that the number of illegal activities is less than the number of states of nature. This situation of incompleteness has been called as fragility of the penal code.

The penal code, in the real world, is quite complex since a crime might be punished by more than two laws. To see how to choose an optimal penal code, we refer to Abreu (1988). However, for the sake of simplicity, we assume that for each crime there is only one law punishing it. Another goal of this paper is the formalisation of the concept of impunity in the market for offenses, using a geometrical approach. This method allows us to price any crime.

To reach our main aims and obtain our findings, we adopt the methodology used in the choice theory in which our decision makers are the criminal individual. These individuals are assumed to be rational in the sense that the criminal are aware of their alternatives, form expectations about unknowns, have clear preferences that in our case are represented by a numerical utility function, and chose their crimes in a deliberate way after some process of optimisation. For the case of pricing of crimes, our methodology is similar to the one used in finance, which in turn comes from the convex analysis theory when we want to separate two convex sets whose intersection is non-empty. To be more precise, we apply a simple separation theorem of convex subsets of a Euclidean space.

Theoretically, our model is just restrict to the supply side. In order to include the demand side we should be able to model the organisations who would demand for crimes. However, by the presence of markets for offenses in the way in which defined them, the objective would be well defined due to fine structure would not be unique\(^4\). On the one part, the multiplicity of the fines would be desirable because it is the judiciary who would choose them, in order to optimize some objective function that benefit the society. However, as the fines in our model are exogenous, we will not be concerned with welfare problems.

Although, we does not carry out any theoretical analysis of general equilibrium, we do offer a short section about equilibrium. In which, by mean an example, we make a static comparative analysis by showing the possible behaviour of the equilibrium allocations when parameters, such as discount factor of criminal, technology of the criminal organisation and expected fines, change.

The paper is organised as follows. Section 2 describes the model. Section 3 offers a geometric interpretation of impunity in the market for offenses. In Section 4, an impunity-free

\(^4\) There would be many ways to discount the futures fines, see Section 4 for more details.
condition is established and our main results are presented, the first is concerned with the pricing of illegal activities and the second one with the rationality of the criminal. In Section 5, we discuss equilibrium properties and some policy implications. Finally, Section 6 concludes.

2. The Model
2.1. The Offense Market

The model has two periods $t = 0, 1$. In the first period $t = 0$ there is no uncertainty while in $t = 1$ there is, which is modelled by a finite set $S = \{1, 2, \ldots, S\}$ of states of nature. There is a finite number $J$ of illegal activities (or crimes). There is only one good for trading in each period and in each state of nature, so that the commodity space is $\mathbb{R}^{S+1}$.

A crime or illegal activity $j$ can be carried out or suffered by any agent. The former is called criminal and the latter victim. Whatever, it is characterised by its first-period return $r_j$ which is exogenously given; and by a random variable $m_j: S \rightarrow \mathbb{R}^+$: The value $m_{js}$ represents the fine applied at state $s$ if the crime $j$ is carried out or the compensation at state $s$ if crime $j$ is suffered.

It is useful to stress that all variables involved as returns, fines, etc. are measured in terms of the only good available in the economy.

Each agent, whether criminal or victim, is characterized by an utility function $u^h: \mathbb{R}^{S+1} \rightarrow \mathbb{R}$ and by an initial endowments $\omega^h \epsilon \mathbb{R}^{S+1}$:

Remark 1: Notice that the agents feel neither pleasure nor remorse when committing crimes. They only engage in illegal activities in order to increase their wealth and therefore their consumption, which does, provided them pleasure. This decision depends only on fines, which are exogenously imposed by the law. Since nobody choose to be victim, our concern in what follow will be focus only in criminal's decisions. That is, when $\theta \epsilon \mathbb{R}^{J}$:

Before describing the criminal's problem, it is convenient to provide some examples of some illegal activities addressed in this paper.

Examples

As said above there are crimes or illegal activities of many forms that ones could imagine. Some require investments, for instance, purchasing of drug for after selling it. Others require a sophisticated technology, which demands high costs or maintenance, for example, gangs involved in counterfeit money or forged documents. Our crimes are simpler, are those which involve no initial investment nor purchasing of equipment for execution. We offer some them: 1) Murders, being $r$ the payment made to the murder for carried out the crime and the number of ordered murders, 2) Drug traffic, being $r$ the payment made to "drug smuggler" and $\theta$ the number units of the drugs trafficked, etc. In these examples, $m_s$ is the fine to be paid by the criminal if she/he is caught. In the case where the offense is non-bailable, the fine can be interpreted as the monetary value lost in the years spent by the Criminal in jail.
2.2. The Criminal’s Problem

Each offender $h$ takes as given the fine structure $M \in R_{+}^{S}$, and chooses a consumption plan $x = (x_o; x) \in R_{+}^{S+1}$ and a crime plan $\theta \in R^{J}$ to be committed in order to maximize his utility function subject to the following budget constraints,

$$x_o \leq \omega_o^h + r \quad (1)$$

And

$$x_s + m_s \theta \leq \omega_s^h; s = 1; \ldots; S. \quad (2)$$

Budget constraint (1) tells us that the consumption expenditure is financed by the value of the criminal's initial endowment plus the value coming from the illegal activities. Budget constraints (2) say that the value of initial endowment in each state of nature finances the contingent consumption and the possible fine, which is also contingent to the states of nature $s \in S$.

Thus the offender $h$'s budget set is defined to be

$$B^h(M) = \{(x; \theta) \in R_{+}^{S+1} x R^{J} : (1) \text{ and } (2) \text{ are satisfied}\}$$

3. Impunity in the Market for Offenses

Impunity means exemption from punishment or loss or escape from fines. Impunity arises from a failure by States to meet their obligations to investigate violations; to respond appropriately in respect of the perpetrators. Particularly, in the area of justice, by ensuring that those suspected of criminal responsibility are prosecuted, tried and duly punished; to provide victims with effective remedies and to ensure that they receive reparation for the injuries suffered; to ensure the inalienable right to know the truth about violations; and to take other necessary steps to prevent a recurrence of violations.

In the next, we will formalize this concept, which would help to the judge, for example to set a fine, to be charged.

**Definition 1. (Unpunished crimes)**

An illegal activity plan $\theta \in R^{J}$ is an unpunished crime if and only if

$$r \theta > 0 \text{ and } M \theta = 0 \quad (3)$$

An illegal activity plan $\theta \in R^{J}$ is a strong unpunished crime if and only if

$$r \theta \geq 0 \text{ and } M \theta < 0 \quad (4)$$

From now, we will simply call unpunished crimes regardless of if they are unpunished or strong unpunished. Formally, we write (1) and (2) in a compact form in the following way:

$$\theta \in R^{J} \text{ is a unpunished crime } \iff ( -r \theta ; M \theta ) \in -R^{I+S}\{0\} \quad (5)$$

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Remark 2: Notice that in definition of impunity of crimes, we are not ruling out the offenders' profit nor the victims' losses because we believe that the fixing of fines on the part of judge should take into account not only the profits of criminals but also the compensation that victims could win. Impunity as defined in (5) allow us to explore its geometry as is shown in the following picture.

Remark 3: Geometrically all the unpunished crimes defined by (1) are located along the negative horizontal line; and the remain of the shaded region below contains all the strong unpunished crimes.

The following definition complete our discussion about impunity in the offenses or more precisely of the legal system of the economy.

Definition 2. It is said that impunity exists for the criminal \( h \) (with respect to the fine structure \( M \)) if he engage\(^5\)s in an illegal activity plan which is an unpunished crime. That is, that satisfies (5).

Definition 3.

1. It is said the structure fine \( M \) is impunity-free if and only if there is no plan of illegal activities which satisfies (5)

2. We say that the legal system is impunity-free (it does not allow impunity) if all fine structures are impunity-free


\(^5\) He can be the offender or the victim.

4. Fundamental Theorem of Crime Pricing

In this Section, we shall use the impunity concept in order to price the crimes. Thus, the following hypothesis called impunity free (IF) will be assumed.

Assumption (IF): The market for offenses is impunity-free.

This assumption says that in societies the following are excluded: first, situations where criminals carry out crimes obtaining strictly positive monetary benefit with no punishment or second, situations where there is a potential for committing crimes ($\pi \theta \geq 0$) and at least in a state of nature there is no punishment.

In order to formalize Assumption IF in a precise manner we need to define the following sub-space,

$$C = \{( - \theta ; M \theta ) \in R^{1+S} : \theta \in R^I \}.$$ 

Therefore, we have the following characterization which follows directly from definitions.

Lemma 1. Assumption IF is satisfied if and only if

$$C \cap ( - R_{+}^{1+S} ) = \{(0; 0)\}.$$ 

We formulate this fundamental theorem in two parts. The first is concerned with the pricing of illegal activities, while the second, (Theorem 2 and Theorem 3 below) deals with the relationship between impunity in the market for offenses and the rationality of the criminal.

4.1. Pricing

Theorem 1. (First Part)

The fine structure $M \in R^{S \times J}$ is impunity-free if and only if there exists $\beta \in R_{+}^{S}$ such that

$$r_j = \sum_{s \in S} \beta_s m_j ; \ \forall j \in J \tag{6}.$$ 

The proof of Theorem 1 is a direct application of Stiemke's Lemma.

Remark 4: It is useful to note $M$ is established by the judiciary and is taken as given by criminals. Hence, Theorem 1, via (6), serves as a guide for judiciary to fix the fines.

Define $\rho = (\rho_1, \ldots, \rho_S) \in [0; 1]$ to be $\rho_s = \frac{\beta_s}{\sum_s \beta_s}$. Thus, is a measure of probability on states of nature $\{1, \ldots, S\}$. Hence, (6) can be written was
Impunity and Rationality in a Market for Offenses

$$r_j = (\sum_\beta \beta_s)E_\beta M^j; \forall j \in J.$$  \hspace{1cm} (7)

**Remark 5:** First of all, the discounted vector $\beta \in \mathbb{R}_{++}^S$ is not necessarily unique unless the number of crimes, $J$ stipulated in the code is equal to the number of the states of nature $S$. If this were the case, we would have a unique manner to determine the expected value of the fine. This follows from (7). However, this is not our case. Second, in order to be able to interpret (7) we can think the factor $\sum_\beta \beta_s$ as a discount factor that judiciary would have to fix for fines. With this in mind, (7) can be paraphrased as: if the legal system is impunity-free, then the value, $r^j$, from illegal activity $j$ must equal to the discounted value of the average fine imposed by the judiciary. Put it in others words, the expected value of the fine is the future value of loot.

4.2. Rationality

It said criminals to be rational if they maximizes their utility functions subject to budget constrains which involves both current return of each crime and the fine structure. The results of this second part are given in two theorems as follows:

**Theorem 2.** (Necessity)

If the utility function is strictly increasing, then non-impunity in the offense markets is a necessary condition for the individual's problem to have a solution.

**Theorem 3.** (Sufficiency)

If the utility function is continuous, then non-impunity in the offense markets is a sufficient condition for the individual's problem to have a solution.

5. Equilibrium

Theorems 1 and 2 are fundamental in order to understand whether the concept of free-impunity is compatible with the orderly function of markets. That is, whether or not in the economy where agents use the offense markets to transfer wealth there exists equilibrium. However, it is useful to point out that the analysis made above is only in relation the supply of crimes. To get a general equilibrium analysis we should be able to model the demand side, in order to construct a model of general equilibrium we need to confront the demand and supply of crimes. In order to do it, the value $r^j$ from crime $j$ can be interpreted as an ex-ante wage received by criminals to commit the crime $j$. This ex-ante wage, $r^j$ should be paid by those demanding crimes, namely the criminal organisation.

In what follows we offer an example to illustrate how equilibrium variables, say demand or supply, change when the parameters such as discounted factors, initial endowments, fine structure do as well.

**Example:** Define $(0.5,0.5)$ to be the common probability assessment of agents on states of nature $S = \{1,2\}$. Set $m = (m_1,m_2)$ for the fine structure and $r$ for the returns of crimes which is interpret as being the ex-ante wage pay for each unit of illegal activity committed. There are two agents. One is a criminal who supplies crimes and the other is a criminal organization who demands crimes.
Supply: Suppose that the criminal is represented by the utility function $U(x_0, x_1, x_2) = \ln(x_0) + \beta/2(x_1 + x_2)$, being $\beta$ is the discount factor. If it is assumed that initial endowments of this criminal is $\omega = (\omega_0, \omega_1, \omega_2)$, then the criminal choose $\theta > 0$ in order to maximize $U$ subject to following budget constraints:

$$x_0 = \omega_0 + r\theta$$
$$x_s = \omega_s - m_s \theta; \ s = 1; 2;$$

Similarly to the case of demand, from the first order conditions we obtain the supply for crimes

$$\theta = \frac{2}{\beta(m_1+m_2)} - \frac{\omega_0}{r} \text{ with } r > M*\beta\omega_0. \quad (9)$$

The criminal will supply crimes if $r$ is greater than the discounted expected fine $M*\beta$, weighted by the first-period initial endowment $\omega_0$.

Demand: Suppose that there is a single criminal organization who demands crimes. It is represented by the returns yield for each $\phi$ demanded. Suppose that these returns are given by the following function $f(\phi) = K \ln(\phi)$.

Given $m = (m_1, m_2)$ the fine structure and $r$ the ex-ante wage, then this organisation must choose $\phi$ in order to maximize,

$$K \ln(\phi) - (r + 0.5(m_1 + m_2)) \phi,$$

where the latter term represents the cost incurred by the amount crime demanded. Using the first order condition, a simple operation yields the demand for crimes.

$$\phi = \frac{K}{r+0.5(m_1+m_2)} \quad \text{(10)}$$

Putting $M*$ for the expected fine $0.5(m_1 + m_2)$ we are going to compute the equilibrium for this special case.

Equilibrium: Equalizing (9) and (10) we get

$$r^2 + (1 - (\omega_0 + K)\beta) M*r - \omega_0 \beta M^2 =0$$

Solving the previous square equation we get $r$ as a function of parameters $\beta, M*$ and $\omega_0$. More precisely we obtain a single positive solution,

$$r^* = \Delta M^*, \quad \text{where}$$

$$\Delta = 0.5 ((1 - (\omega_0 + K)\beta)^2 + 4\omega_0 \beta)^{1/2} - (1 - (\omega_0 + K)\beta))$$

Using $r^*$, we can compute the criminal behavior represented by $\phi^* = \theta^*$.

Putting $r^*$ into (10) we get

$$\phi^* = \frac{K}{\Delta + 1 M^*}$$

Where $K$ can be interpreted as the degree of the development, that criminal organisation has.

Remark 6: First, it is easy to verify that in equilibrium the supply is an increasing function of $K$ and a decreasing function of the expected fine $M*$. Second, the supply of equilibrium as well
equilibrium demand converges to $1/(M^*\beta)$ as $K$ tends to infinity. Third, this means although the society has a criminal organisation well developed the amount for crimes do not explode. Forth, by increasing the expect value of the fines, the policy maker could decrease the amount of crimes in the society, and lastly notice that to increase the value of the fines does not necessarily result in an increase their expected values, policymakers need to modify the criminal subjective probabilities. One possible way to get this goal, for some crimes, is to increase the number of police officer on the streets, since as some authors have found there is a negative correlation between the number of police officers and criminal rates for some crimes\textsuperscript{6}.

5.1. Numerical Analysis

In what follows, we present a numerical analysis of the result presented in the section above. In the below graphics we consider $M^* = 2$ and $\beta=0.94$. First, in Fig. (2) we see the behaviour of the demand and supply equilibrium functions when we vary $K$. We can observe graphically the increasing and convergence behavior of the equilibrium allocations for different levels of initial endowments.

![Figure 2: Demand and Supply.](image)

Now in Fig. (3) We present the shape for the same demand and supply equilibrium function when we vary $K$ and the discount factor, for the same levels of initial endowment considered above.

The numerical analysis of sensitivity in relation to expected fine is straightforward since the supply is inversely proportional to it, i.e., a hyperbola.

\textsuperscript{6} See Levitt (2002).
6. Conclusion

In this paper, we have analysed the behaviour of criminals under the assumptions of incompleteness and non-impunity of the market for offenses. We have formalised the concept of non-impunity and obtained a kind of fundamental theorem of crime-pricing in the markets for offenses. In addition, we have conducted an equilibrium analysis and studied its properties. Concluding that policymaker must carry any effort to increase the expected value of the fines, which can be achieved by law enforcements, increasing the number of police officers on the streets and reducing the judicial costs.

Interesting issues are the analysis of general equilibrium existence and to what extent the presence of such crimes are Pareto improvements, we left these issues for future research.

Appendix

Proof of Theorem 2

Proof. Suppose that there is impunity in the offense markets so that there exists a plan of activities \( \bar{o} \) where under the fine structure \( M \) the following conditions

\[
(r \bar{o} \geq 0 \text{ and } M \bar{o} < 0) \text{ or } (r \bar{o} > 0 \text{ and } M \bar{o} = 0 ),
\]

are satisfied.

Let \((x_o; x; \theta) \in \mathbb{R}_+^{S+1} \times \mathbb{R}^{l} \) be a feasible crime-consumption plan. Then, the two following inequalities are true:

\[
x_o + r\bar{o} \leq \omega_o + r(\bar{o} + \theta);
\]

\[
(x_s-M\bar{o}) + M(\theta + \bar{o}) \leq \omega_s.
\]

From the definition of \( \bar{o} \) it follows that the resulting consumption plan \((x_o + r\bar{o}, x_s-M\bar{o})\) is strictly preferred to \((x_o; x_s)\) because the criminal's function is strictly increasing.

Therefore, the criminal's problem has no solution, which is a contradiction. Thus, the market for offenses is impunity-free.

Figure 3: Demand and Supply for any \( \beta \).
Proof of Theorem 3

Proof. Because of the continuity of the utility function, it is sufficient to show the compactness of the criminal's budget set. Clearly, the budget set is closed. It is therefore sufficient to prove that it is bounded. In fact, multiplying inequality (2) by $\beta_s$ and adding in $s$ we have:

$$\sum_s \beta_s (x_s + m_s) \leq \sum_s \beta_s \omega_s.$$  

Adding this last inequality with (1) and using Theorem 1, one has

$$x_o + \sum_s \beta_s x_s \leq \omega_o + \sum_s \beta_s \omega_s,$$

which implies that $(x_o, x)$ is bounded, since the initial endowment is assumed to be non-zero. Next, we prove that $\theta$ belonging to the budget set is bounded. Suppose this is not true. Then, there will be a sequence $\theta^n$ satisfying (1) and (2) such that $||\theta^n|| \to \infty$, as $n$ goes to infinity. Thus the sequence $\{\theta^n/||\theta^n||\}_{n\in N}$ is bounded and belongs to the unitary sphere, $S^{J-1}$, of $R^J$. Since $R^I$ is a metric space, such a sequence will admit a convergent subsequence\textsuperscript{7}.

On the other hand, we notice that the consumption $(x_o; x) \in R_J S^{J+1}$ is positive, then from (1) and (2) one has the following,

$$0 \leq x_o \leq \omega_o + r \theta^n \to r \theta^n \geq \omega_o,$$

$$m_s \theta^n \leq x_s + m_s \theta^n \leq \omega_s, \quad s \in S.$$

Let $\theta \in S^{J-1}$ be the limit of the previous subsequence. Then, taking limit when $n$ tends to infinity, in the previous inequalities one has that:

$$M \theta = 0.$$

Incompleteness ($J < S$) implies that the linear transformation $M: R^J \to R^S$ is injective. Thus, its kernel consists only of the null vector. So, $\theta = 0$.

However, this contradicts the fact $\theta$ belongs to $S^{J-1}$. Therefore the original sequence $\{\theta^n\}$ is bounded, which implies that the budget set is compact, concluding the proof.

7. References


\textsuperscript{7} which, without loss of generality we denote as the same sequence.


